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DIFFUSION FLAME ANALYSIS OF TWIN PLANE JETS VIA A KINETIC THEORY APPROACH

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SUMMARY

Diffusion flame solutions of twin plane jets based on a turbulent kinetic theory due to Chung and a Green function method by Hong are presented. The chemical reaction between fuel and oxidizer is assumed to be one-step, one-direction and infinitely fast. The solutions are obtained by direct integration over a constructed probability density function in velocity space. The probability density functions of reactants in transverse velocity space, species mass fraction distributions, turbulent transport of momentum and heat, temperature distributions and flame structure are also considered in this paper. The diffusion flame phenomena of the twin plane jets show that the interaction between the two jets is a dominant factor.

KEY WORDS Twin plane jets Diffusion flame Turbulent kinetic theory Turbulent heat transfer

INTRODUCTION

Multiple-jet flows are widely used in engineering and industry, e.g. in thrust-augmenting ejectors in aircraft, furnaces and combustors in industry, etc. Studies of the reactive flow of twin plane jets provide not only information on the basic structure of multiple-jet reactive flows but also important guidelines for the design of combustors with multiple jets.

The phenomenological theory approach to turbulence modelling is unable to account for the fundamentals of the turbulent diffusion flame structure.[1–3] At present we are attempting an alternative approach to turbulence modelling for solving turbulent reactive flows. It is a kinetic theory approach which uses a probability density function (PDF) to characterize the statistical structure in velocity space without involving the turbulent transport coefficients or eddy viscosities.[4] Hong and Wang[5] have investigated the turbulent diffusion flame of a plane jet via the PDF model. The aim of this paper is to use the turbulent kinetic theory approach due to Chung[6] to analyse the diffusion flame of twin-jet turbulent flow.

Most approaches to turbulence modelling have been developed from the concept of gradient-type transport. The present work is an alternative approach to turbulence modelling which seeks a PDF to characterize the stochastic characteristics of fluid elements in velocity space. It is a kinetic theory approach. Since 1967 a number of turbulent kinetic theories have been proposed, such as those of Lundgren,[7,8] Fox,[9,10] Yen[11] and Haworth and Pope.[12–14] A Langevin model appropriate to constant-property turbulent flows was developed from the general equation for

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the fluid particle velocity increment proposed by Pope.[15] The objective of Pope's work was to determine the form of a second-order tensor appearing in the general model equation as a function of local mean quantities. A modelled transport equation for the joint probability density function (JPDF) of the velocities and a scalar has been solved numerically for four self-similar turbulent free shear flows.[12] In this context, modifications to the Langevin equation have been made, both to the deterministic term and to the form of the random term.[16,17] These theories were limited to studies of flows without chemical reaction. A more general kinetic theory of turbulence for chemically reacting flows was developed by Chung.[6,18-21] Chung's equation was solved by Hong[22] using a Green function method to directly integrate the equation for the PDF. This method has been successfully modified for free shear layer mixing and combustion problems.[23] In the modified Green function method the authors actually solved an instantaneous mixing problem to simulate the steady state phenomenon. This study[23] demonstrated the inherent advantages of the Green function method for solving Chung's kinetic turbulence equations. Thus the present treatment of the diffusion flame of turbulent twin jets using the Green function method and the assumed combustion model for the constructed PDF are warranted.

Early experimental work on the twin-jet problem was carried out by Miller and Comings.[24] Other researchers[25-27] investigated similar problems. However, those investigations were limited to measurements of the lower-order mean quantities. In Reference 4 we calculated results for twin-plane-jet turbulent mixing via Chung's kinetic theory approach. Although the higher-order correlations and transported correlations were given in that paper, the study was also limited to flows without chemical reaction. Once the PDF is obtained and the combustion model is assumed, one can proceed to calculate the properties of twin-plane-jet flow with chemical reaction; the species PDF distributions, species mass fraction distributions, temperature distributions and flame structure are given in the present analysis. From the present calculated results the diffusion flame structure of twin plane jets can be better understood via the kinetic theory approach. The cross-correlation behaviour of the fluctuations also shows that the phenomenon of turbulent transport is different from that of average-gradient type transport (i.e. phenomenological theory).

THEORETICAL MODEL

Figure 1 shows the flow field configuration of the twin plane jets in the present problem. It consists of two jet streams which carry the fuel, while the surrounding stream carries the oxidant and diluent. The chemical reaction is assumed to be one-step and one-direction, i.e.

$$f+g \xrightarrow{k_r} p, \tag{1}$$

where f (fuel), g (oxidant) and p (products) represent the number of moles of each species. Dimensionless variables h, m and n are defined as

$$\begin{aligned} h&=(C_p/\Delta h_0)C_t,\\ m&=(pM_p/fM_r)C_r,\\ n&=(pM_p/gM_s)C_s. \end{aligned} \tag{2}$$

The instantaneous production rate of combustion products is given by

$$\omega_p=k_0\exp(-\Delta E/RC_t)C_r^f C_s^g, \tag{3}$$

where k_0 is a constant and ΔE, R, C_t, ω_p, C_r^f and C_s^g represent activation energy, gas constant,

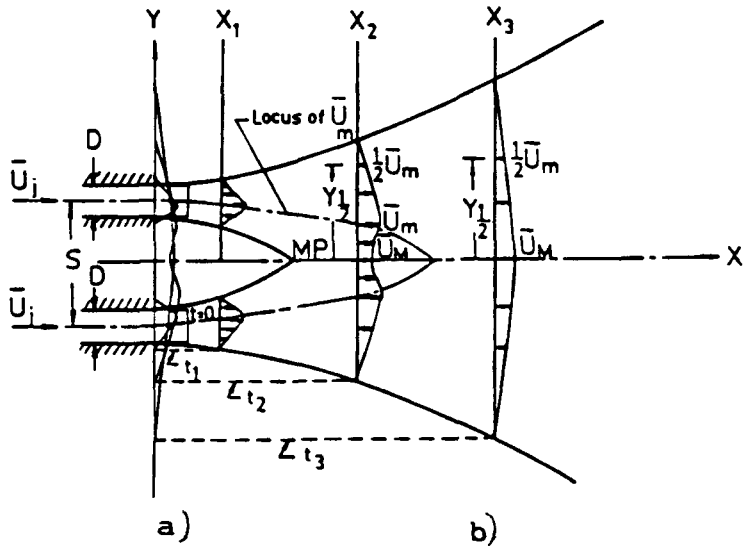

Figure 1. Simulation of (a) instantaneous twin-jet mixing as (b) steady twin-jet mixing

absolute temperature, chemical production rate and mean mass fraction of fuel and oxidant respectively. The instantaneous production rates of fuel, ω_r, oxidant, ω_s, and energy, ω_t, due to chemical reaction are related to ω_p.

Chung's kinetic equation with chemical reaction[6] is written as

$$\frac{\partial F_l}{\partial t}+u_j\frac{\partial F_l}{\partial x_j}=\beta\frac{\partial}{\partial u_j}[(u_j-\bar{u}_j)F_l]+\frac{\beta_1}{3}E\frac{\partial^2 F_l}{\partial u_j\partial\bar{u}_j}+\omega_l f, \tag{4}$$

where

$$F_l=C_l f, \qquad \beta=\beta_1+\beta^v, \qquad \Sigma F_l=f\Sigma C_l=f.$$

The full set of equations is represented by equation (4) with

$$l=\begin{cases} \text{r, fuel,}\\ \text{s, oxidant,}\\ \text{t, temperature,}\\ \text{p, product}\\ \text{c, inert gas.}\end{cases}$$

Equations (1)–(4) govern the combustion of reactants in the twin plane jets. Both $(F_m)_i$ and $(F_n)_i$ are further defined to represent the values of F_m and F_n respectively without chemical reaction; then

$$(F_m)_i-(F_n)_i=F_m-F_n. \tag{5}$$

If the chemical reaction rate is infinite, i.e. $k_0\to\infty$ (or $k_f\to\infty$), application of this limit to equations (2) and (3) results in

$$F_m F_n=0. \tag{6}$$

Under the restriction of an infinite chemical reaction rate, the physical meaning of equation (6) is that the chemical reaction can proceed when fuel and oxidant appear simultaneously in the velocity space. Therefore the ability of the reactants to react depends upon their history in the turbulence field. One important difference between the laminar and turbulent flame is that F_m and F_n are functions of the independent velocity variable. The physical explanation for the turbulent diffusion flame of the present statistical model of the flame zone will be apparent when the solutions are discussed. Equation (6) will be used to manage the reactive flow of the twin plane jets.

Now, if one does not consider the chemical reaction, so that $C=1$ and $\omega=0$, then equation (4) can be simplified as

$$\frac{\partial F}{\partial t}+(k_j-\beta u_j)\frac{\partial F}{\partial u_j}+u_j\frac{\partial F}{\partial x_j}=3\beta F+q_1\frac{\partial^2 F}{\partial u_j\,\partial u_j}, \tag{7}$$

where

$$k_j\equiv\beta\bar{u}_j, \tag{8}$$

$$q_1\equiv\tfrac{1}{3}\beta_1 E. \tag{9}$$

The solution of equation (7) is Green's function.[22]

In the present analysis we simulate the transient phenomenon shown in Figure 1(a) as the steady twin-plane-jet mixing shown in Figure 1(b).[4,5,23] In order to use the results of momentum without chemical reaction, C_2H_6 is used as the fuel because its density almost the same as that of air. The chemical reaction involved is

$$C_2H_6+\tfrac{7}{2}O_2\xrightarrow{k_f}2CO_2+3H_2O. \tag{10}$$

Before mixing, the source conditions for the PDFs of the average velocities $\bar{U}_j$ and $\bar{U}_\infty$ are Gaussian distributions because the jets and their surroundings undergo nearly laminar flow. Therefore the source conditions for the scalar C can be written as

$$S_{0i}=C_{i0}f_{0i}(\mathbf{u}_0), \tag{11}$$

$$f_{01}=\frac{1}{(\frac{2}{3}\pi E_{01})^{3/2}}\exp\left(-\frac{(u_0-\bar{U}_\infty)^2+v_0^2+w_0^2}{\frac{2}{3}E_{01}}\right),$$

$$f_{0i}=\frac{1}{(\frac{2}{3}\pi E_{0i})^{3/2}}\exp\left(-\frac{(u_0-\bar{U}_j)^2+v_0^2+w_0^2}{\frac{2}{3}E_{0i}}\right),\quad i=2,3. \tag{12}$$

According to the above-mentioned physical problem, the source conditions for the mass fractions of the reactants can be written as

$$C_{i0}=C_{r0}=1{\cdot}0\ (i=2,3),\ -(S+D)/2\leqslant Y\leqslant-(S-D)/2,\quad (S-D)/2\leqslant Y\leqslant(S+D)/2,\quad x\leqslant 0, \tag{13}$$

$$C_{i0}=\begin{cases}C_{s0}=0{\cdot}22\\ C_{c0}=0{\cdot}78\end{cases}(i=1),\quad Y\leqslant-(S+D)/2,-(S-D)/2\leqslant Y\leqslant(S-D)/2,\quad Y\geqslant(S+D)/2,x\leqslant 0. \tag{14}$$

From equations (2) and (10),

$$m=\tfrac{142}{30}C_r,\qquad n=\tfrac{142}{112}C_s. \tag{15}$$

Therefore the source conditions m_0 and n_0 for m and n respectively become

$$\begin{cases} m_0 = 1{\cdot}0 \times \frac{142}{30}, & -(S+D)/2 \leqslant Y \leqslant -(S-D)/2, \quad (S-D)/2 \leqslant Y \leqslant (S+D)/2, \quad x \leqslant 0, \\ n_0 = 0, \end{cases} \tag{16}$$

$$\begin{cases} m_0 = 0, & Y \leqslant -(S+D)/2, \quad -(S-D)/2 \leqslant Y \leqslant (S-D)/2, \quad Y \geqslant (S+D)/2, \quad x \leqslant 0. \\ n_0 = 0{\cdot}22 \times \frac{142}{112}, \end{cases} \tag{17}$$

According to the above source conditions, one can obtain the constructed PDF of twin-jet flow as

$$F = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} GS_{01}\,dy_0 - \int_{-(S+D)/2}^{-(S-D)/2} GS_{01}\,dy_0 - \int_{(S-D)/2}^{(S+D)/2} GS_{01}\,dy_0 + \int_{-(S+D)/2}^{-(S-D)/2} GS_{02}\,dy_0 + \int_{(S-D)/2}^{(S+D)/2} GS_{03}\,dy_0\right) du_0\,dv_0\,dw_0\,dx_0\,dz_0. \tag{18}$$

The integration of equation (18) is very tedious.[4] From the constructed PDF the ensemble average $\langle Q \rangle$ can be obtained as

$$\langle Q \rangle = \int_{-\infty}^{\infty} FQ\,d\mathbf{u}. \tag{19}$$

Let $Q = u_i' C_j$ and $u_i' T'$; then we can obtain the corresponding relation of turbulent correlations. In the present problem the major mixing direction between fuel and oxidant is the y-direction, so equation (6) can be simplified as

$$F_{m,v} F_{n,v} = 0, \tag{20}$$

where

$$F_{m,v} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_m(\mathbf{u})\,du\,dw, \qquad F_{n,v} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_n(\mathbf{u})\,du\,dw. \tag{21}$$

Equation (20) represents the PDF in velocity space, which was simplified to the v-direction only. That is, the chemical reaction can occur only when fuel and oxidant simultaneously appear in v-velocity space. The v-direction velocity space was further divided into v_m and v_n, i.e.

$$\begin{aligned} &(F_{m,v})_i \geqslant (F_{n,v})_i, \quad v = v_m, \\ &(F_{m,v})_i \leqslant (F_{n,v})_i, \quad v = v_n, \end{aligned} \tag{22}$$

where

$$(F_{m,v})_i \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (F_m)_i\,du\,dw, \qquad (F_{n,v})_i \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (F_n)_i\,du\,dw. \tag{23}$$

Therefore equations (5) and (21) can be replaced by

$$\begin{aligned} F_{m,v} &= (F_{m,v})_i - (F_{n,v})_i \\ F_{n,v} &= 0 \end{aligned} \quad (v = v_m), \tag{24}$$

$$\begin{aligned} F_{n,v} &= (F_{n,v})_i - (F_{m,v})_i \\ F_{m,v} &= 0 \end{aligned} \quad (v = v_n). \tag{25}$$

Equations (24) and (25) are the final results for the present combustion model of twin jets with reactive flow. The source conditions are given by equations (16) and (17).

RESULTS AND DISCUSSION

Species PDFs in velocity space

$RF_{m,v'}$ and $RF_{n,v'}$ are defined as

$$RF_{m,v'} = \tfrac{30}{142} F_{m,v'}, \qquad RF_{n,v'} = \tfrac{112}{142} F_{n,v'},$$

where $RF_{m,v'}$ and $RF_{n,v'}$ represent the PDFs of fuel and oxidant in v'-velocity space respectively. Solutions of $RF_{m,v'}$ and $RF_{n,v'}$ are shown in Figures 2–4. The PDF distributions of fuel are located in the positive and negative v'-velocity space because the mixing flow comes from twin jets (up/down directions with respect to the centre line). The PDF distributions of ambient fluid are located in the neighbourhood of $v'=0$ owing to the lower velocity. From the calculated results of species PDF distributions we can infer that the effects of the reactive flow on the local turbulence properties are due to the interaction of two different eddy characteristics.

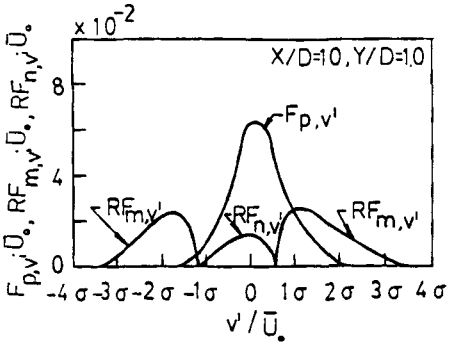

Figure 2. Species PDFs ($X/D=10$, $Y/D=1$)

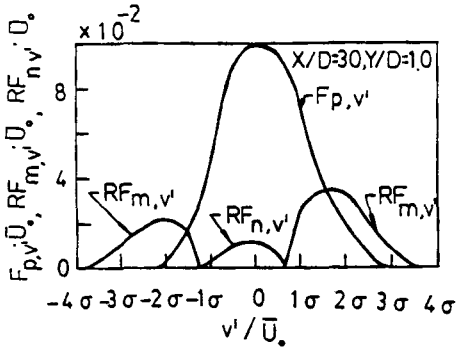

Figure 3. Species PDFs ($X/D=30$, $Y/D=1$)

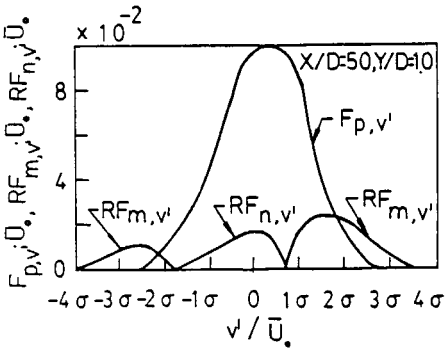

Figure 4. Species PDFs ($X/D=50$, $Y/D=1$)

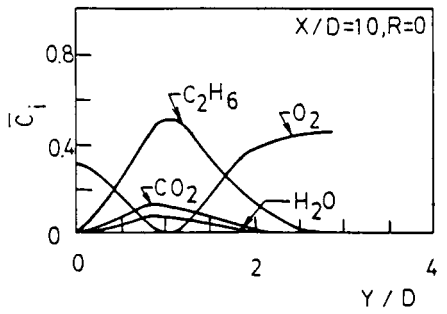

Figure 5. Mass fraction distributions ($X/D=10$, $R=0$)

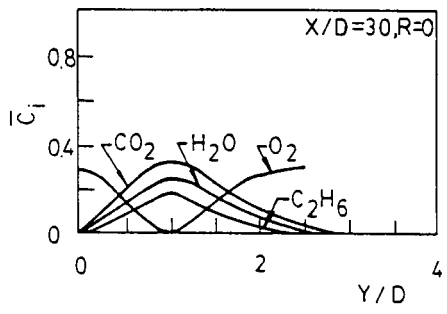

Figure 6. Mass fraction distributions ($X/D=30$, $R=0$)

Mass fraction distributions

The mass fraction distributions as a function of cross-section are shown in Figures 5–7. Under the present combustion model, the reactants are only brought into coexistence by the history of velocity fluctuations space experienced by each reactant of the combustion process. The mean profiles of fuel and oxidant are overlapped as expected. Products are formed in the overlapped region, defined as the flame zone. The mean profiles for the present theory are free from the discontinuous slopes associated with earlier approximate solutions.[20] In the B-region[4] of the

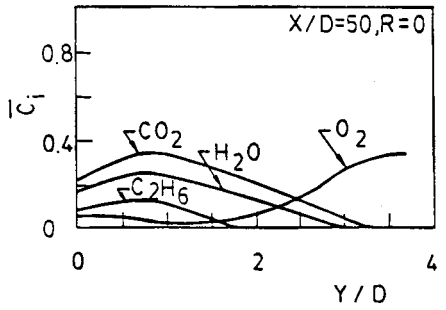

Figure 7. Mass fraction distributions ($X/D=50$, $R=0$)

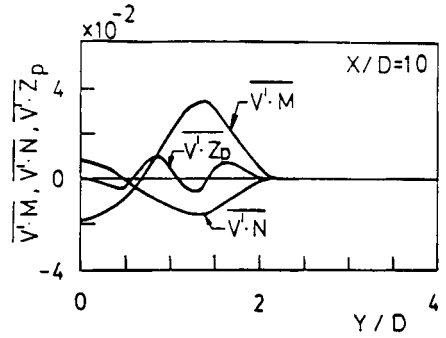

Figure 8. Turbulent transport of mass fraction in v'-velocity space ($X/D=10$)

initial converging region there a two flame zones which have come from the two different turbulence length scales of flow, as shown in Figures 5 and 6. In the combined region the two turbulence length scales are coupled into a single turbulence length scale resulting from the interaction of turbulent mixing between the twin jets, as shown in Figure 7. Therefore the flame zones of the combined region become a single flame zone. The fundamental nature of the solutions shows that the thickness of the turbulent flame zone is of the order of the turbulence length scale.

Turbulent transport

The mass fraction transport as a function of species in v'-velocity space is shown in Figures 8–10. For the B-region[4] ($X/D=10$) the distributions of fuel mass fraction transport are shown in Figure 8, where positive distributions represent fuel transport outwards and negative distributions represent fuel transport inwards. The fuel transport is zero at the approximate location $Y/D=0{\cdot}75$. In contrast, in Figure 5 one finds that the maximum species mass fraction of the fuel is at $Y/D=1{\cdot}0$, i.e. there exists a small section ($0{\cdot}75 \leqslant Y/D \leqslant 1{\cdot}0$) where the mass fraction is transported up the gradient. This demonstrates that the turbulent eddy transport is different from the average-gradient-type transport (i.e. phenomenological theory). Along the downstream the mass fraction transport of species recovers gradually to the single-jet type[5] because the flow field approaches the combined region, as shown in Figures 9 and 10. The distributions of turbulent heat transfer $\langle v'T'\rangle$ ($\approx K_t \partial\langle T\rangle/\partial\langle Y\rangle$) as a function of cross-section are shown in Figures 11–13. In the B-region[4] the turbulent heat transfer reaches a maximum for a cross-section $X/D=30$

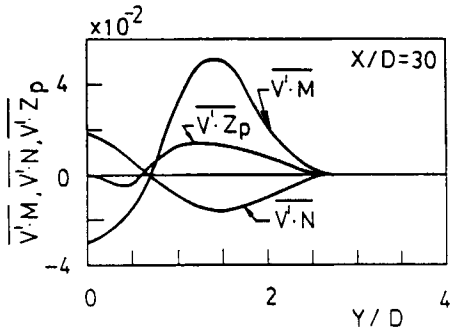

Figure 9. Turbulent transport of mass fraction in v'-velocity space ($X/D=30$)

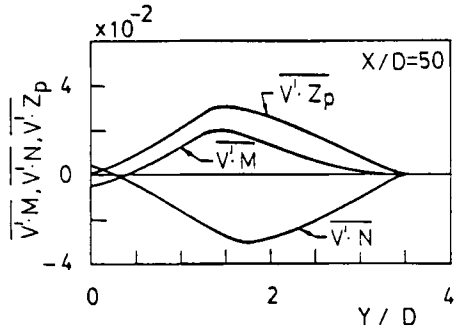

Figure 10. Turbulent transport of mass fraction in v'-velocity space ($X/D=50$)

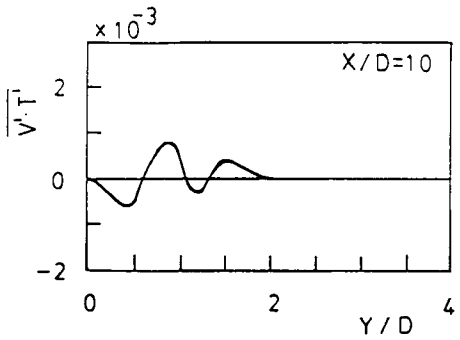

Figure 11. Turbulent transfer of heat in v'-velocity space ($X/D=10$)

owing to the strong turbulent eddy transport. From Figures 11–13 one can see that the turbulent heat transfer of twin-jet flow is affected by the eddy mixing and interaction between the twin jets. One finds that the turbulent transport and the molecular transport are obviously different phenomena, as seen by comparing Figures 11–13 with Figure 14. There also exists a section where the turbulent heat transfer is transported up the gradient.

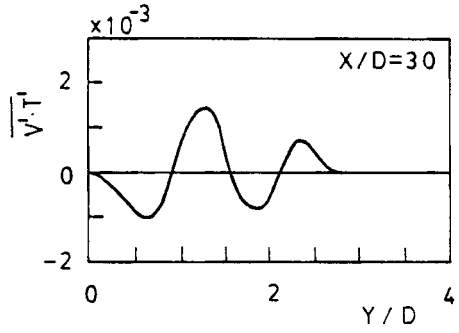

Figure 12. Turbulent transfer of heat in v'-velocity space ($X/D=30$)

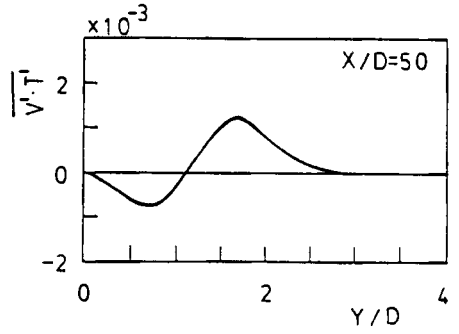

Figure 13. Turbulent transfer of heat in v'-velocity space ($X/D=50$)

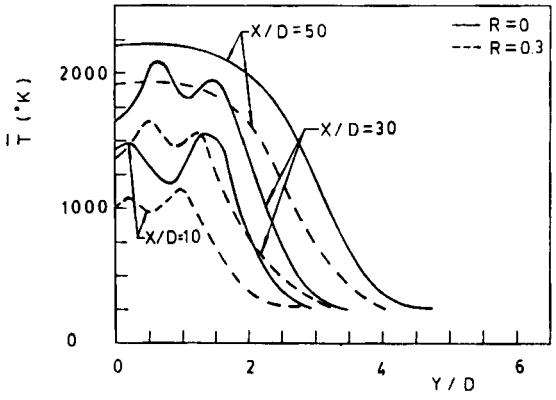

Figure 14. Distributions of average temperature

Temperature distributions

The distributions of average temperature are shown in Figure 14. In the B-region (i.e. $X/D=10$–30) the two 'peak' phenomena are formed by the two flame zones resulting from the interaction of the two different eddy characteristics. The interaction between the different eddies plays an important role in the temperature distributions.

Diffusion flame structure

In the present combustion model the flame zone is defined only by the coexistence of oxidant and fuel in velocity space. In this paper the flame zone is constructed when the mass fraction of fuel is not less than 10^{-4}. The twin-jet diffusion flame structures for $R=0$ and 0·3 are shown in Figures 15 and 16. These flame structure are compared with the flame structures of a single jet obtained by Hong and Wang,[5] as shown in Figures 17 and 18. The difference in flame structure between the twin jets and the single jet is the inner diffusion flame zone in the twin jets. This inner diffusion flame zone results from the interaction of the inner flow between the two jets. The length of the outer diffusion flame for the twin jets is larger than that for the single jet[5] owing to the interaction of the twin jets. The shear stress and eddy transport are dominated in effect by the velocity ratio R.[5] Therefore the flame length of flow is affected by the velocity ratio. In general, the surrounding fluid of the combustor is not stationary, so the velocity ratio is an important factor in combustor design. The eddy transport is small when the velocity ratio is increased owing to the small velocity difference between nozzle exit and surroundings. Therefore the combustion process proceeds far away for a small velocity ratio. The outer flame length for $R=0$ is about $X/D=240$ and for $R=0{\cdot}3$ is about $X/D=420$, as shown in Figures 15 and 16. This phenomenon is the same

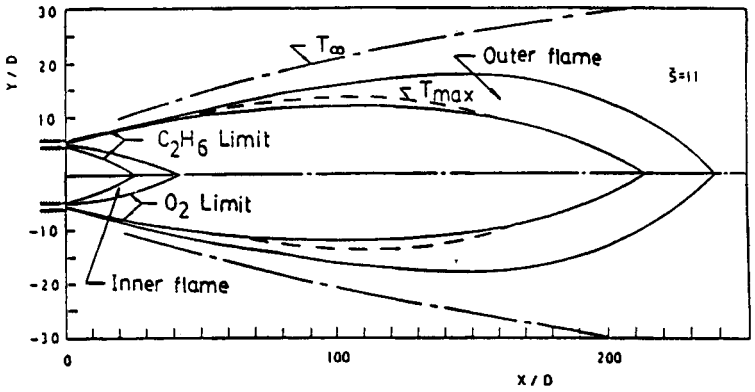

Figure 15. Flame structure ($n=2$, $R=0$)

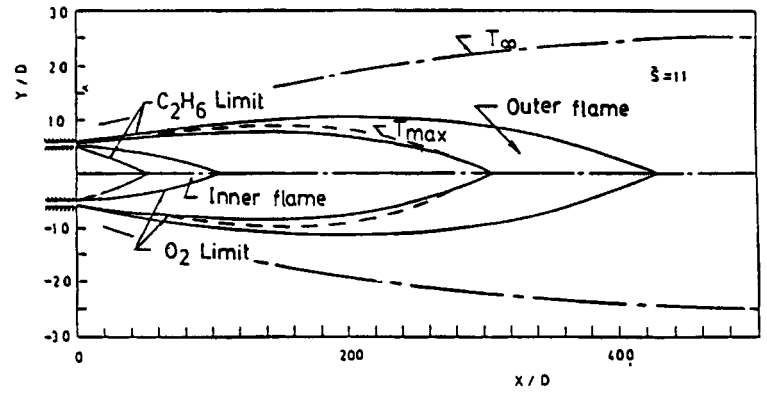

Figure 16. Flame structure ($n=2$, $R=0{\cdot}3$)

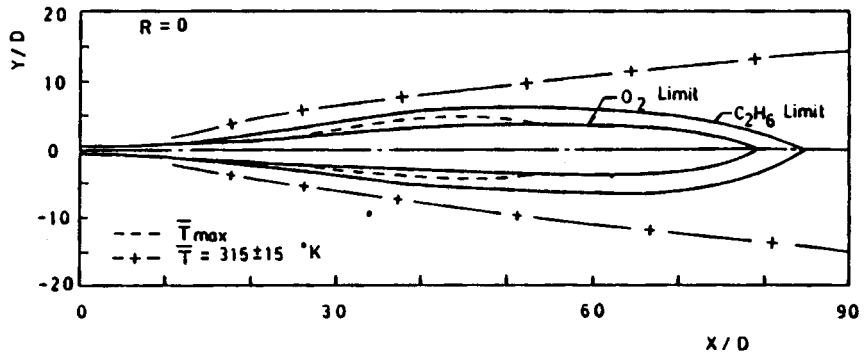

Figure 17. Flame structure[5] ($n=1$, $R=0$)

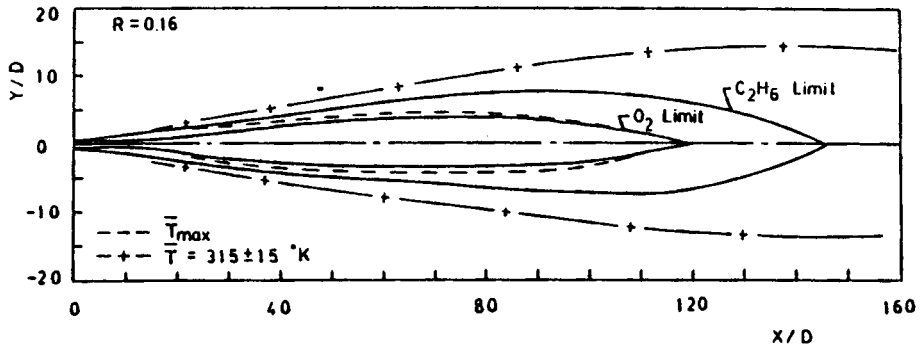

Figure 18. Flame structure[5] ($n=1$, $R=0{\cdot}16$)

as for the single jet.[5] The thermal effect region in this paper is defined as $T=315\pm15$ K. For $X/D=200$ the width of the thermal effect region at $R=0$ ($60D$) is larger than at $R=0{\cdot}3$ ($38D$) because the shear stress for $R=0{\cdot}3$ is smaller than for $R=0$.

CONCLUDING REMARKS

Diffusion flame solutions of twin jets based on the kinetic theory approach of Chung and the Green function solution of Hong have been presented. The species probability density functions, species mass fractions, turbulent transport of momentum and heat, average temperatures and flame structures have been calculated. The effects of the species PDF distributions on the diffusion flame are shown to play an important role. The development of the diffusion flame for twin jets can be better predicted via the PDF distributions of species. The turbulent diffusion flame of the twin-jet flow shows an inner diffusion flame zone which is absent from the flame of the single jet. The present analysis shows that the eddy transport of turbulent reactive flow is completely different from the molecular transport of laminar reactive flow. The flame zone within the model only occurs when the reactants coexist in the same velocity space. The existence of a thick diffusion flame within the model, even with an infinitely fast reaction rate, coincides with the experimental results of Hawthorne *et al.*[28] Some inside structure of engineering problems for any

higher-order turbulent transport phenomena of twin jets with reactive flow can be predicted and explained via the kinetic theory of turbulence approach.

APPENDIX: NOMENCLATURE

C	mass fraction of concentration
C_p	specific heat at constant pressure
D	width of jets
E	turbulent energy ($E=\overline{u'^2}+\overline{v'^2}+\overline{w'^2}$)
ΔE	activation energy
F	probability density function
f	PDF of fluid element
f_0	source condition for PDF of fluid element
$f(u'), f(v'), f(w')$	PDF of fluctuation velocity in x-, y- and z-direction respectively
G	Green's function
Δh_0	formulation heat of chemical reaction
k_f	specific reaction rate ($k_f=k_0\exp(-\Delta E/RC_t)$)
M	molecular weight
n	number of jets
R	gas constant or velocity ratio ($\bar{U}_\infty/\bar{U}_j$)
$RF_{m,v'}$	PDF of fuel in v'-direction velocity space
$RF_{n,v'}$	PDF of oxidant in v'-direction velocity space
S	space between jets
S_0	source condition at $x=0$
T	average temperature
T'	fluctuation temperature
t	time
$\bar{U}_j$	velocity of jets at nozzle exit
$\bar{U}_M$	mixed centre velocity
$\bar{U}_0$	relative velocity ($\bar{U}_0=\bar{U}_M-\bar{U}_\infty$)
$\bar{U}_\infty$	velocity of surroundings
$\bar{U}, \bar{V}, \bar{W}$	mean velocity in x-, y- and z-direction respectively
u, v, w	instantaneous velocity in x-, y- and z-direction respectively
u', v', w'	fluctuation velocity in x-, y- and z-direction respectively
X	co-ordinate of jet axis
Y	co-ordinate of y-axis
β_1	characteristic relaxation rate of energy-containing eddies
β^v	characteristic relaxation rate of microscale
ω	chemical reaction term
Λ_1	characteristic scale of energy-containing eddies

Subscripts and superscripts

$\bar{}$ or $\langle\ \rangle$	ensemble average
$\rightarrow$	vector
∞	free stream condition
0	source condition
c	inert gas

f	fuel
g	oxidant
i	jet number
j	tensor
p	product
t	temperature

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